Instructions: Complete each of the following exercises for practice.

1. Find the limit (if it exists), or show it does not exist.

   (a) $\lim\limits_{(x,y)\to(3,2)}(x^2y^3-4y^2)$

   (b) $\lim\limits_{(x,y)\to(2,-1)}\dfrac{x^2y+xy^2}{x^2-y^2}$

   (c) $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^4-4y^2}{x^2+2y^2}$

   (d) $\lim\limits_{(x,y)\to(0,0)}\dfrac{5y^4\cos^2(x)}{x^4+y^4}$

   (e) $\lim\limits_{(x,y)\to(0,0)}\dfrac{xy}{\sqrt{x^2+y^2}}$

   (f) $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^3-y^3}{x^2+xy+y^2}$

   (g) $\lim\limits_{(x,y)\to(0,0)}\dfrac{y^2\sin^2(x)}{x^4+y^4}$

   (h) $\lim\limits_{(x,y,z)\to(\pi,0,\frac{1}{3})}e^{y^2}\tan(xz)$

   (i) $\lim\limits_{(x,y,z)\to(0,0,0)}\dfrac{xy+yz}{x^2+y^2+z^2}$

2. Compute $h(x,y)=g(f(x,y))$ and find the set of all points at which h is continuous.

   (a) $g(t)=t^2+\sqrt{t},\quad f(x,y)=2x+3y-6$

   (b) $g(t)=t+\ln(t),\quad f(x,y)=\dfrac{1-xy}{1+x^2y^2}$

3. Determine the set of points at which the function is continuous.

   (a) $f(x,y)=\dfrac{1+x^2+y^2}{1-x^2-y^2}$

   (b) $g(x,y)=\dfrac{e^x+e^y}{e^{xy}-1}$

   (c) $h(x,y)=\ln(1+x-y)$ $k(x,y)=\arcsin(x^2+y^2+z^2)$

4. Use polar coordinates to compute the limit (**Hint**: As $(x,y)\to(0,0)$ you can always have $r\to 0^+$...).

   (a) $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^3+y^3}{x^2+y^2}$

   (b) $\lim\limits_{(x,y)\to(0,0)}(x^2+y^2)\ln(x^2+y^2)$

   (c) $\lim\limits_{(x,y)\to(0,0)}\dfrac{e^{-x^2-y^2}-1}{x^2+y^2}$

   (d) $\lim\limits_{(x,y)\to(0,0)}\dfrac{\sin(x^2+y^2)}{x^2+y^2}$

5. Prove that the function $f(\mathbf{x})=\mathbf{v}\cdot\mathbf{x}$ is continuous on $\mathbb{R}^n$ for all $\mathbf{v}\in\mathbb{R}^n$.

6. Prove that the function $f(\mathbf{x})=|\mathbf{x}|$ is continuous on $\mathbb{R}^n$. (**Hint**: What type of function is $|\mathbf{x}|^2$?)